The 2018 International Conference On Control Automation & Information Sciences (ICCAIS 2018)
October 24-27, 2018, Hangzhou, China.

Multistatic Doppler-based marine ships tracking

Cong-Thanh Do
School of Civil and Mechanical Engineering
Curtin University
WA 6102, Australia
thanhdc.dldktd@gmail.com

Hoa Van Nguyen
School of Computer Science
The University of Adelaide
SA 5005, Australia
hoavan.nguyen@adelaide.edu.au

***Abstract*—Multistatic Doppler radar measurements for multiple targets are typically corrupted by noise, missed detections and false alarms. In addition, when targets are close together, it becomes more difficult for the tracker to resolve tracks due to the highly nonlinear and low observability nature of the observation. This paper proposes a solution to the problem of tracking multiple marine ships from multistatic Doppler measurements. We use close form labeled multitarget Bayes filter, which can accommodate unknown and time-varying number of targets, clutter misdetection and association uncertainty. The efficiency of the proposed algorithm is illustrated via numerical simulation examples.**

***Index Terms*—Random Finite Set, Multitarget Tracking, Multistatic Doppler measurement.**

I. INTRODUCTION

In both civilian applications and modern electronic warfare, radar-based surveillance plays a key role in detecting and tracking targets for not only prevention but also interception strategies. Doppler radar is a type of passive radar, which has several advantages in this domain, such as light weight, compact design, low power consumption, and not easily be detected. Multistatic Doppler radar system, which contains multiple spatially distributed static radars, have been used for tracking multiple targets with higher accuracy [1], [2].

In practice there are several challenges in the use of multistatic Doppler radars. Firstly, there are generic challenges in multitarget tracking such as unknown and randomly time-varying number of targets, detection uncertainty, clutter and data association uncertainty [3]. Secondly, multistatic Doppler radar involves multiple sensors, nonlinearity and low observability of the Doppler measurement [1], [2], which compounds the difficulty in data association.

Three main approaches have been developed to address the multitarget tracking problem, namely: the Joint Probabilistic Data Association Filter (JPDAF); Multiple Hypothesis Tracking (MHT); and Random Finite Set (RFS) [3]. While JPDAF and MHT involve modifying single target tracking filters to deal with multiple targets, the RFS approach provides a top-down formulation based on fundamental concepts in estimation theory, including multitarget estimation error [4] and Bayes optimality [5], [6].

This paper is conducted under the support of Curtin International Postgraduate Research Scholarship - Ministry of Education and Training Vietnam (CIPRS-MOET).

The RFS approach has gained world-wide interest in recent years, and is recognised as the future method of choice in multitarget tracking [7]. Since its inception, a suite of multitarget filters have been developed, e.g. the Probability Hypothesis Density (PHD) filter [8], [9], Cardinalized PHD filter [10], [11], multi-Bernoulli filters [4], [12], Generalized Labeled Multi-Bernoulli (GLMB) or Vo-Vo filter [13], [14][1], Labeled Multi-Bernoulli filter [15], and multi-scan GLMB filter [16].

RFS techniques have been successfully deployed in many applications from tracking and data fusion [6], to machine learning [17]. RFS-based filters such PHD/CPHD and multi-Bernoulli have been applied to computer vision [18]–[20], robotics [21] automotive safety [22], sensor scheduling [23]–[28], and sensor network [29], [52], [53]. The Vo-Vo filter outputs target tracks and can be implemented with linear complexity in the number of measurements using Gibbs sampling [30]. Furthermore, it has been applied to tracking from merged measurements [31], extended targets [32], maneuvering targets [33], track-before-detect [34]–[36], computer vision [37], [38], sensor scheduling [39], [40], distributed fusion [41], [42], field robotics [43], and cell microscopy [44].

Although many RFS-based filters such as the PHD and multi-Bernoulli filters have been applied to Doppler measurements [45], [46], they do not produce target tracks and are crude approximations to the Bayes multitarget filter. Being one of the most efficient multitarget tracker, demonstrated to track over a million targets per scan [47], the Vo-Vo filter is therefore, most suitable to address the challenges of tracking multiple marine vessels in a timely manner.

Apart from the introduction, the paper comprises the following four sections. First, the background of target dynamic model with multistatic Doppler measurement, and the labeled RFS approach to multitarget tracking is presented. Second, the Vo-Vo filter applied to the tracking of marine ships with multistatic Doppler measurements is shown. Third, validation of the proposed solution is illustrated by numerical simulations, followed by some concluding remarks.

II. BACKGROUND

A. Target dynamic model

We consider a nonlinear multitarget sceriano with several marine ships whose kinematic state includes latitude-longitude

[1]For short we use the term Vo-Vo filter coined by Mahler in [6].

The 2018 International Conference On Control Automation & Information Sciences (ICCAIS 2018)
October 24-27, 2018, Hangzhou, China.

position $p_k = [\mu_k, \lambda_k]^T$, speed $\nu_k = [\dot{\mu}_k, \dot{\lambda}_k]^T$ and the course ψ, i.e. $\tilde{x}_k = [x_k^T, \psi]^T$ where $x_k = [\mu_k, \dot{\mu}_k, \lambda_k, \dot{\lambda}_k]^T$. Note that, latitude, longitude and course are measured in degree (°), while distance, velocity and time are calculated in nautical mile (M), knot (kn), and hours (h), respectively. Each target is modeled by a Gaussian Markov transition density $f_{k|k-1}(x_k|x_{k-1}) = \mathcal{N}(x_k; \hat{x}_k, \mathbf{R}_k)$, where $\hat{x}_k = [[\mathbf{F}(\psi_k)x_{k-1}]^T, \psi_k]^T, \mathbf{R}_k = diag([\sigma_v^2 \mathbf{G}_k \mathbf{G}_k^T, \sigma_\psi])$,

$$\mathbf{F}(\psi) = \begin{bmatrix} 1 & \frac{\sin(\Delta\psi)}{\psi} & 0 & \frac{(\cos(\Delta\psi)-1)}{\psi} \\ 0 & \cos(\Delta\psi) & 0 & -\sin(\Delta\psi) \\ 0 & -\frac{(\cos(\Delta\psi)-1)}{\psi} & 1 & \frac{\sin(\Delta\psi)}{\psi} \\ 0 & \sin(\Delta\psi) & 0 & \cos(\Delta\psi) \end{bmatrix}; \mathbf{G}_k = \begin{bmatrix} \frac{\Delta^2}{2} & 0 \\ \Delta & 0 \\ 0 & \frac{\Delta^2}{2} \\ 0 & \Delta \end{bmatrix}, \tag{1}$$

Δ is the sampling period, σ_v is the standard deviation of the process noise, σ_ψ is the standard deviation of the course noise.

Remark 1: The target model with transition matrix $\mathbf{F}(\psi)$ given in (1) is based on the assumption that the surveillance area is not located near the North/South Poles.

B. Measurement model

In this work, we consider a multistatic passive Doppler radar system consisting of one cooperative transmitter and two spatially distributed receivers (see Fig.1). This system measures the speed of the target at a distance by bouncing pulses of radio signals off the target, collecting the signals reflected from the target then analyzing the altered frequency of returned signals based on the Doppler effect [48]. The Doppler measurement of a target with state x_k at the sth receiver is given by

$$z_k^{(s)} = -\nu_k^T \left(\frac{p_k - p_r^{(s)}}{\left\| p_k - p_r^{(s)} \right\|} + \frac{p_k - p_t}{\| p_k - p_t \|} \right) \frac{f_t}{c} + w_k. \tag{2}$$

in which the position p_k and speed ν_k have been defined in the previous part, $p_t = [\mu_t, \lambda_t]^T$ is the transmitter location, and $p_r^{(s)} = [\mu_r^{(s)}, \lambda_r^{(s)}]^T$ is the sth-receiver position; w_k is zero-mean Gaussian noise with covariance $\mathbf{Q}$, $w_k \sim \mathcal{N}(0, \mathbf{Q}_k)$; and f_t and c are the emitted signal frequency of the transmiter and the speed of light, respectively.

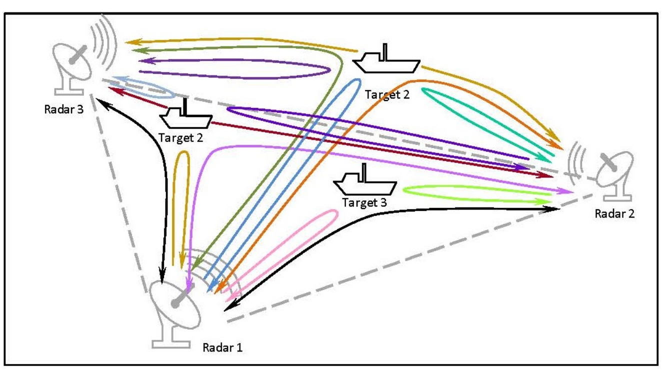

Fig. 1: Multitarget tracking by using Doppler radar sceriano.

Remark 2: Since the target can move towards or aways from the receiver/transmitter, the observation equation (2) based on Doppler effect can be negative or positive in the known interval $[-f_0, +f_0]$ of the Doppler sensor. It is obvious that both state and measurement equations are highly nonlinear.

C. Multitarget State and Labeled RFS Models

In a multitarget system, both the states and the number of the targets randomly vary with time. The objective of multitarget tracking is to jointly estimate not only the number of targets and their states but also the trajectories of the targets via information collected from sensors.

For simplicity, the following notations will be used throughout the paper. An unlabeled single target state is denoted by lower-case leters (i.e. x, y, z, etc.), and an unlabeled multitarget state (a finite set of single states) is represented by upper-case leters (X, Y, Z,etc.) while the labeled states are denoted by bold-faced letters ($\mathbf{x}, \mathbf{X}, etc.$). The spaces these corresponding variables are denoted by blackboard bold letters ($\mathbb{X}, \mathbb{L}, \mathbb{Z}, etc$). Let $1_S(\cdot)$ denotes the inclusion function of a given set S, $1_S(X) = 1$ if $X \subseteq S$ and zero otherwise, the $\mathcal{F}(S)$ represents the class of finite subsets of S. In addition, let $\langle f, g \rangle = \int f(x)g(x)dx$ denotes the inner product, $f^X = \prod_{x \in X} f(x)$ (with $f^\emptyset = 1$), and the generalized Kronecker delta function δ_Y receiving arbitrary sets, vectors, intergers, etc., as its arguments, i.e. $\delta_Y[X] = 1$ if $X = Y$, and zero otherwise.

Following [13], [14], we identify the target kinematic states by assigning a distinct label ℓ_i drawn from a discrete label space $\mathbb{L}_k$ to each state x_i at time k, that is:

$$\mathbf{X}_k \triangleq \{\mathbf{x}_1, \cdots, \mathbf{x}_n\} = \{(x_1, \ell_1), \cdots, (x_n, \ell_n)\} \subseteq \mathbb{X}_k \times \mathbb{L}_k \tag{3}$$

where $\mathbb{L}_k$ is given recursively by the union of the label space $\mathbb{L}_{k-1}$ of the targets born prior to time k and the label space $\mathbb{B}_k$ of the targets born at time k, $\mathbb{L}_k = \mathbb{L}_{k-1} \cup \mathbb{B}_k$. Then the distinct labels provide the means to identify the tracks of individual targets. Denote the function which extracts the labels of the labeled set $\mathbf{X}_k$ given in (3) as $\mathcal{L}(\mathbf{X}) = \{\ell_1, \ldots, \ell_n\}$, then for a labeled set with distinct labels we have $|\mathcal{L}(\mathbf{X})| = |\mathbf{X}|$, where $|\mathbf{X}|$ denotes the cardinality of the set $\mathbf{X}$. Thus we define the distinct label indicator function as $\Delta(\mathbf{X}) \triangleq \delta_{|\mathbf{X}|}[|\mathcal{L}(\mathbf{X})|]$.

Definition 1: [13] A labeled multi-Bernoulli (LMB) RFS is a labeled RFS with state space $\mathbb{X}$ and discrete label space $\mathbb{L}$, which follows the probability distribution

$$\pi(\mathbf{X}) = \Delta(\mathbf{X}) \omega(\mathcal{L}(\mathbf{X})) \left[p(\cdot) \right]^{\mathbf{X}} \tag{4}$$

where

$$\begin{aligned} \omega(L) &= \prod_{i \in \mathbb{L}} (1 - r^{(i)}) \prod_{\ell \in \mathbb{L}} \frac{1_L(\ell) r^{(\ell)}}{1 - r^{(\ell)}} \\ p(\mathbf{x}) &= p^{(\ell)}(x) \end{aligned} \tag{5}$$

in which $r^{(\ell)}$ and $p^{(\ell)}(\cdot)$ are the existence probability and probability density corresponding to label $\ell \in \mathbb{L}$.

Definition 2: [13] A generalized labeled multi-Bernoulli (GLMB) RFS is a labeled RFS with state space $\mathbb{X}$ and discrete label space $\mathbb{L}$, which satisfies the probability distribution

$$\pi(\mathbf{X}) = \Delta(\mathbf{X}) \sum_{c \in \mathbb{C}} w^{(c)}(\mathcal{L}(\mathbf{X})) \left[p^{(c)}(\cdot) \right]^{\mathbf{X}} \tag{6}$$

978-1-5386-6020-1/18/$31.00 ©2018 IEEE

where $\mathbb{C}$ is an arbitrary index set, and

$$\sum_{L\subseteq\mathbb{L}}\sum_{c\subseteq\mathbb{C}} w^{(c)}(L) = 1$$
$$\int_{x\in\mathbb{X}} p^{(c)}(x,\ell)dx = 1 \tag{7}$$

III. The Vo-Vo Filter

A. Multitarget System Model

Suppose that the multitarget state $\mathbf{X}_{k-1}$ at time *k-1* encapsulates N target states $\mathbf{x}_{k-1,i}, i = 1,\ldots,N$, then the multitarget state $\mathbf{X}_k$ at time k is the union of all surviving and new birth targets

$$\mathbf{X}_k = \bigcup_{\mathbf{x}_{k-1}\in\mathbf{X}_{k-1}} \mathbf{S}_{k|k-1}(\mathbf{x}_{k-1})\bigcup B_k.$$

Given $\mathbf{x}_{k-1}$, the state at the next time is modelled by a labeled Bernoulli RFS $\mathbf{S}_{k|k-1}(\mathbf{x}_{k-1})$. The set B_k of new-born states is modelled by a labeled multi-Bernoulli RFS, defined as follows.

A labeled state $\mathbf{x} = (x,\ell) \in \mathbf{X}$ can either exists and evolves to a new state $\mathbf{x}_+$ at the next time step with survival probability $P_S(\mathbf{x})$ and probability density $f(x_+|x,\ell)\delta_\ell(\ell_+)$ (where $f(x_+|x,\ell)$ is the single target transition kernel), or disappears with probability $1-P_S(\mathbf{x})$. Following [13], [14], the set $\mathbf{S}$ of surviving targets at the next time is distributed as

$$f_S(\mathbf{S}|\mathbf{X}) = \Delta(\mathbf{S})\Delta(\mathbf{X})1_{\mathcal{L}(\mathbf{X})}(\mathcal{L}(\mathbf{S}))\left[\mathbf{\Phi}(\mathbf{S};\cdot)\right]^{\mathbf{X}} \tag{8}$$

where

$$\begin{aligned}\mathbf{\Phi}(\mathbf{S};x,\ell) = &\sum_{(x_+,\ell_+)\in\mathbf{S}} \delta_\ell(\ell_+)P_S(x,\ell)f(x_+|x,\ell)\\ &+(1-1_{\mathcal{L}(\mathbf{S})}(\ell))(1-P_S(x,\ell))\end{aligned} \tag{9}$$

The expression for the multi-object transition density is given by

$$f(\mathbf{X}_+|\mathbf{X}) = f_S(\mathbf{X}_+\cap(\mathbb{X}\times\mathbb{L})|\mathbf{X})f_B(\mathbf{X}_+-(\mathbb{X}\times\mathbb{L})) \tag{10}$$

where $f_B(\cdot)$ is the distribution of labeled multi-Bernoulli RFS of newborn targets with label space $\mathbb{B}$ (see [13], [14], [31] for more details).

Given a multitarget state $\mathbf{X}_{k-1}$, the multitarget measurement Z_{k-1} is the union of Bernoulli RFS $\Theta_{k-1}(\mathbf{x}_{k-1})$ for every $\mathbf{x}_{k-1}\in\mathbf{X}_{k-1}$ and the set K_{k-1} of Poisson clutter or false alarms

$$Z_{k-1} = \bigcup_{\mathbf{x}\in\mathbf{X}_{k-1}}\Theta_{k-1}(\mathbf{x})\bigcup K_{k-1}.$$

Each state $\mathbf{x} = (x,\ell)\in\mathbf{X}$ at time k is either detected by the sth receiver with detection probability $P_D^{(s)}(\mathbf{x})$ and generates an observation $z^{(s)}$ with likelihood $g^{(s)}(z^{(s)}|\mathbf{x})$ or missed with probability $1-P_D^{(s)}(\mathbf{x})$. The multitarget observation at time k, $Z^{(s)} = \{z_1^{(s)},\ldots,z_{M^{(s)}}^{(s)}\}$, is the superposition of the observations from detected objects and Poisson clutter with intensity $\kappa^{(s)}$, false alarms. Assuming that, conditional on $\mathbf{X}$, detections are independent of each other and clutter, the multitarget likelihood is given by [13], [14]

$$g^{(s)}(Z^{(s)}|\mathbf{X}) = e^{-\langle\kappa^{(s)},1\rangle}(\kappa^{(s)})^K\sum_{\theta\in\Theta}\left[\Psi_{Z^{(s)}}^{(s)}(\mathbf{x};\theta)\right]^{\mathbf{X}} \tag{11}$$

where $e^{-\langle\kappa^{(s)},1\rangle}(\kappa^{(s)})^K = \pi_K^{(s)}(K)$ is the distribution density funtion of the set *K*, $\theta:\mathbb{L}\to 0,1,\ldots,|Z^{(s)}|$, such that $[\theta(i) = \theta(j) > 0] \Rightarrow [i=j]$ (meaning that each measurement is assigned to at most 1 target). The function $\Psi_Z^{(s)}(\cdot;\theta)$ is given by

$$\Psi_Z^{(s)}(\mathbf{x};\theta) = \begin{cases}\frac{P_D^{(s)}(\mathbf{x})g^{(s)}(z_{\theta(\ell)}^{(s)}|\mathbf{x})}{\kappa^{(s)}(z_{\theta(\ell)}^{(s)})}, & \theta(\ell)>0\\ 1-P_D^{(s)}(\mathbf{x}) & \theta(\ell)=0.\end{cases} \tag{12}$$

The multi-sensor multitarget likelihood at time *k*, is given by

$$g_k(Z_k|\mathbf{X}_k) = \prod_{s=1}^N g_k^{(s)}\left(Z_k^{(s)}|\mathbf{X}_k\right) \tag{13}$$

where $Z_k = \left(Z_k^{(1)},\ldots,Z_k^{(s)},\ldots,Z_k^{(N)}\right)$ is *N* sensor observation set.

B. Multitarget Bayes tracking Filter

The multitarget Bayes filter can be written in the form of Chapman-Kolmogorov equation and Bayes rule as follows [5], [6]:
Predict:

$$\pi_{k|k-1}(\mathbf{X}_k|Z_{1:k-1}) = \int f_{k|k-1}(\mathbf{X}_k|Z_{k-1})\pi_{k-1}(\mathbf{X}_{k-1}|Z_{1:k-1})\delta\mathbf{X},$$

Update:

$$\pi_k(\mathbf{X}_k|Z_{1:k}) = \frac{g_k(Z_k|\mathbf{X}_k)\pi_{k|k-1}(\mathbf{X}_k|Z_{1:k-1})}{\int g_k(Z_k|\mathbf{X})\pi_{k|k-1}(\mathbf{X}|Z_{1:k-1})\delta\mathbf{X}},$$

The above recursion shows how the posterior (or filtering) density $\pi_k(\mathbf{X}_k|Z_{1:k})$ is computed from the prior density $\pi_{k-1}(\mathbf{X}_{k-1}|Z_{1:k-1})$, state transition density $f_{k|k-1}(\mathbf{X}_k|Z_{k-1})$ and measurement likelihood $g_k(Z_k|\mathbf{X}_k)$. The multitarget transition encapsulates the underlying models of target motions, births and deaths, while the multitarget likelihood combines the underlying models of detections and false alarms.

The Vo-Vo filter is a recursion for progagating the parameters of the GLMB density forward in time when new observation arrives. This recursion can be expressed as two-stage procedure [13]:

1) Update : The posterior multitarget density based on the standard multitarget observation likelihood function (11) is given by

$$\pi^{(s)}(\mathbf{X}|Z^{(s)}) = \Delta(\mathbf{X})\sum_{c\in\mathbb{C}}\sum_{\theta\in\Theta(\mathcal{L}(\mathbf{X}))}\omega_{Z^{(s)}}^{(c,\theta)}(\mathcal{L}(\mathbf{X}))\left[p^{(c,\theta)}(\cdot|Z^{(s)})\right]^{\mathbf{X}}, \tag{14}$$

where

$$\begin{aligned}\omega^{(c,\theta)}_{Z^{(s)}}(L) =& \frac{\omega^{(c)}(L)\left[\bar{\Psi}^{(s,c)}_{Z^{(s)}}\right]^{J}}{\sum_{Y\subseteq\mathbb{L}}\sum_{c\in\mathbb{C}}\sum_{\theta\in\Theta(Y)}\omega^{(c)}(Y)\left[\bar{\Psi}^{(s,c)}_{Z^{(s)}}\right]^{Y}}\\ p^{(c,\theta)}(x,\ell|Z^{(s)}) =& \frac{p^{(c)}(x,\ell)\Psi^{(s)}_{Z}(x,\ell;\theta)}{\bar{\Psi}^{(s,c)}_{Z^{(s)}}(\ell)}\\ \bar{\Psi}^{(s,c)}_{Z^{(s)}}(\ell) =& \langle p^{(c)}(\cdot,\ell),\Psi^{(s)}_{Z}(\cdot,\ell;\theta)\rangle,\end{aligned}$$

$\Psi^{(s)}_{Z}(x,\ell;\theta)$ is given in (12).

2) Prediction: Given a Vo-Vo filtering density (6) at time k, the prediction density to time $k+1$ is given by [13]

$$\pi_{+}(\mathbf{X}) = \Delta(\mathbf{X})\sum_{c\in\mathbb{C}}\omega^{(c)}_{+}(\mathcal{L}(\mathbf{X}))\left[p^{(c)}_{+}(\cdot)\right]^{\mathbf{X}} \tag{15}$$

where

$$\begin{aligned}\omega^{(c)}_{+}(L) =& \omega_{B}(L\cap\mathbb{B})\omega^{(c)}_{S}(L\cap\mathbb{L}),\\ p^{(c)}_{+}(x,\ell) =& 1_{\mathbb{L}}(\ell)p^{(c)}_{S}(x,\ell)+1_{\mathbb{B}}(\ell)p_{B}(x,\ell)\\ p^{(c)}_{S}(x,\ell) =& \frac{\langle P_{S}(x,\ell)f(x|\cdot,\ell),p^{(c)}(\cdot,\ell)\rangle}{\gamma^{(c)}_{S}(\ell)},\\ \bar{p}^{(c)}_{S}(\ell) =& \langle P_{S}(\cdot,\ell)f(x|\cdot,\ell),p^{(c)}(\cdot,\ell)\rangle\\ \omega^{(c)}_{S}(Y) =& [\bar{p}^{(c)}_{S}]^{Y}\sum_{J\subseteq\mathbb{L}}1_{J}(Y)\left[q^{(c)}_{S}(\ell)\right]^{J-Y}\omega^{(c)}(J)\\ q^{(c)}_{S}(\ell) =& \left[\langle 1-P_{S}(\cdot,\ell),p^{(c)}(\cdot,\ell)\rangle\right].\end{aligned}$$

C. Implementations

The number of terms in a Vo-Vo updated multitarget density grows exponentially with time. Hence it is not practical to exhaustively evaluate all terms. It is possible to use the K-short path algorithm to compute the best terms of the prediction. However, unless we keep a large number of predicted terms, those with targets births are very few, which could lead to tracking loss. To mitigate this problem, via unscented trasformation, the predictions for target births and survivals are performed separately and then combined afterwards [14].

Measurement gating and prunning are used in the Vo-Vo filter to reduce the number of computations in the update stage. For the single-sensor case, two techniques have been proposed to perform the truncation without having to propagate all the components: Murty's ranked assignment algorithm and Gibbs sampling [14], [30]. Murty's algorithm can be used to determine a given number of highest weighted components of the multi-object filtering density without exhausitively generating all possible mappings, while Gibbs sampler can generate the significant components of the multi-object filtering density for large number of targets to be tracked.

Since the problem of tracking with Doppler measurements in this work requires multiple sensors, the extension of the Gibbs sampler implementation to multiple sensor proposed in [49] is most appropriate. The implementation of the two sensor Vo-Vo filter developed in [50] using Murty's algorithm has a cubic complexity in the product of the number of measurements from the sensors. As a proof of concept of how the Vo-Vo filter addresses multistatic Doppler measurements, the simpler "iterated corrector" implementation that apply single-sensor updates once for each sensor in turn has been used. This strategy would yield the exact solution if all components of the multitarget filtering density are kept.

IV. Numerical studies

In the present work, a total of 10 birth-and-death time-varying marine ships, and misssed-detection-and-clutter observations are considered. The target transition matrix is given by equation (1), in which

$$\Delta = 0.12(\mathrm{h});\quad \sigma_{v} = 0.27(\mathrm{M\,h^{-1}}),\quad \sigma_{\psi} = \pi/180(\mathrm{rad\,s^{-1}}).$$

The survival probability for the target is $P_{S,k}(x) = 0.95$. The birth process is assumed as labeled Poisson RFS with equal intensity $\beta_{k}(x) = \sum_{i=1}^{4} w^{(i)}_{\beta}\mathcal{N}(x;\hat{x}^{(i)}_{\beta},\mathbf{P}_{\beta})$ where all common existence probabilities $w^{(i)}_{\beta} = 0.15$; the mean position values are assumed to be

$$\begin{aligned}\hat{x}^{(1)}_{\beta} =& [17.20°N,0,110.7°E,0,0]^{T}\\ \hat{x}^{(2)}_{\beta} =& [14.60°N,0,113.0°E,0,0]^{T}\\ \hat{x}^{(3)}_{\beta} =& [17.20°N,0,113.0°E,0,0]^{T}\\ \hat{x}^{(4)}_{\beta} =& [18.30°N,0,107.7°E,0,0]^{T},\end{aligned}$$

and

$$\mathbf{P}_{\beta} = diag([3.5'N,26(\mathrm{kn}),3.5'E,26(\mathrm{kn}),6\pi/180(\mathrm{rad/s})])^{2}.$$

Ten targets are assummed to be distributed around the birth model with the closest and farthest latitude distances being 2.85 km and 10.73 km, and the corresponding values for longitute distances being 2.6 km and 10.6 km respectively. The absolute velocities of the targets are assumed to be varied from 2 to 32 kn (approximate 3.5km/h to 60km/h).

The positions of the multistatic passive Doppler radar transmitter p_{t} and receivers $p^{(i)}_{r}, (i = 1,2)$ are assumed to be located at

$$p_{t} = [15°22'58.82''N, 109°07'11.52'']^{T}$$

with transmit frequency $f_{t} = 900\mathrm{MHz}$;

$$\begin{aligned}p^{(1)}_{r} =& [10^{0}22'31.28"N, 114^{0}28'13.45"E]^{T},\\ p^{(2)}_{r} =& [17^{0}58'41.87"N, 106^{0}24'23.98"E]^{T}\end{aligned}$$

with the measurement space for each receiver $[-200\mathrm{Hz}, 200\mathrm{Hz}]$; the measurement noise w_{k} is zero-mean Gaussian noise with covariance $\mathbf{Q}_{k} = diag([1\mathrm{Hz}^{2}, 1\mathrm{Hz}^{2}])$; and c is the speed of light, respectively. The probability of detection is state dependent and is given by

$$p_{D,k}(x) \propto \mathcal{N}(x,[-200\mathrm{Hz}\quad 200\mathrm{Hz}],[-200\mathrm{Hz}\quad 200\mathrm{Hz}])^{2}$$

with peak value of $p_{D,k}(x) = 0.98$. Clutter follows a Poisson RFS with an average rate of $\lambda_{c} = 25$.

Both multitarget scenario with total ten targets appearing randomly in numbers, postitions, and velocities conducted in the surveillance region $[10°N\quad 30°N; 100°E\quad 125°E]$ and the corresponding estimates are shown in Fig.2. The tracker can exactly-track the targets in latitude and longitude with respect to the time (Fig.3). There are some delays and misdetection in observations, which may be due to the distances from the estimated positions and the actual positions. The tracker shows the effectiveness of tracking algorithm when the targets are merged or close together.

Optimal Sub-Pattern Assignment (OSPA) metric is a distance between two sets of points that jointly accounts for the dissimilarity in number of points and the values of the points in the respective sets. OSPA-on-OSPA, or OSPA$^{(2)}$, distance has different interpretation to that of the traditional OSPA distance.

By using both Optimal SubPattern Assignment (OSPA) [51] and OSPA$^{(2)}$ [47] with cut-off parameter c = 100, and p =1, the tracking errors of the filters are evaluated and compared as illustrated in Fig.4. The hidden meaning for using both $OSPA$ and $OSPA^2$ in this paper is not only capturing the errors between the true and estimated multi-target states at the current time step only (via OSPA), but also capturing the error between the true and estimated sets of tracks over a period of time steps (via OSPA$^{(2)}$).

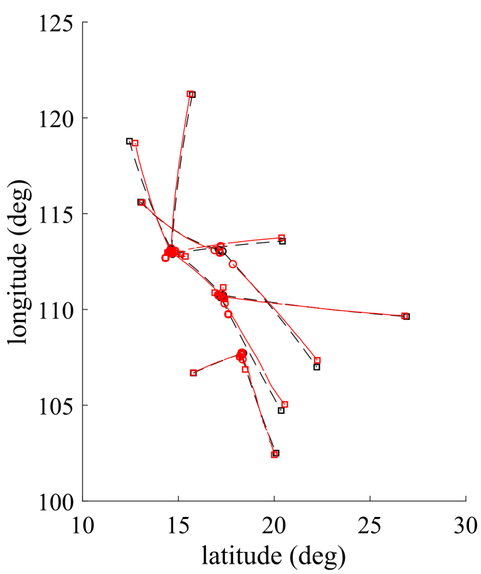

Fig. 2: True tracks (dash-lines) and corresponding etimates (solid lines) in latitude-longitude coordinates. Start/Stop positions for each track are illustrated by ○/□.

V. Conclusion

This paper presented an efficient solution to the problem of tracking an unknown and time varying number of marine ships using multistatic Doppler measurements. In particular, the iterated corrector version of the Vo-Vo filter was applied to the problem to address Doppler measurements from multiple sources. To the best of our knowledge, this is the first study to apply principled online algorithms for tracking marine ships with multistatic Doppler measurements.

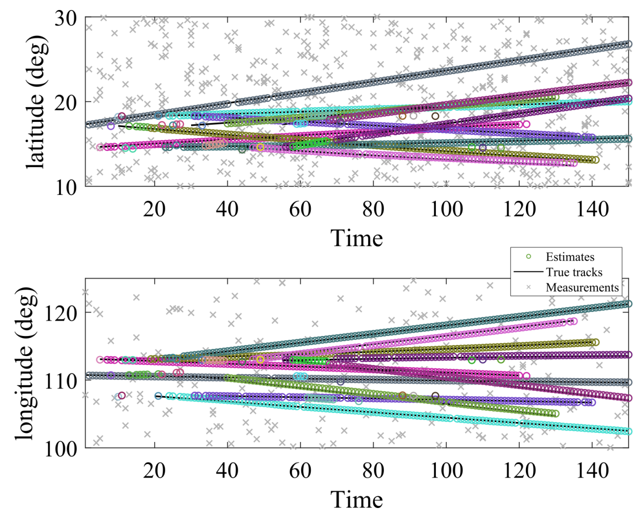

Fig. 3: Estimates and track for latitude and longitude plane versus time.

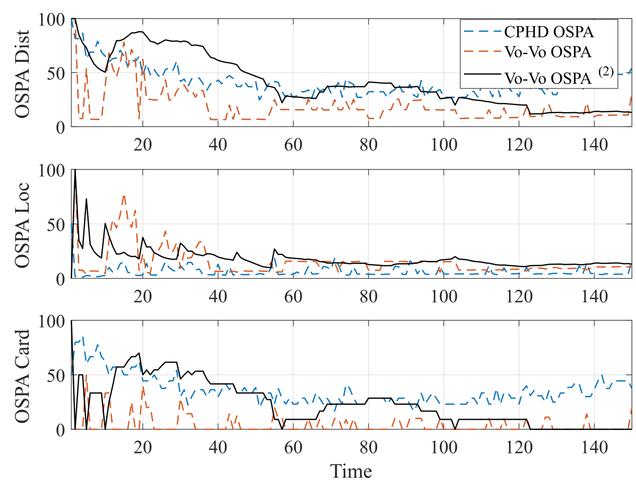

Fig. 4: The OSPA errors of distance, localization and cardinality estimation

References

[1] N. A. Goodman, D. Bruyere, , "Optimum and decentralized detection for multistatic airborne radar, " *IEEE Trans. Aerosp. Electron. Syst.*, vol 43, no. 2, 2007.

[2] G. E. Smith, K. Woodbridge, C. J. Baker, and H. Griffiths, "Multistatic micro-Doppler radar signatures of personnel targets, " *IET Signal Process.*, vol. 4, no. 3, pp. 224 - 233, 2010.

[3] B.-N. Vo, M. Mallick, Y. Bar-Shalom, S. Coraluppi, R. Osborne III, R. Mahler, and B.-T. Vo, "Multitarget Tracking," *Wiley Encyclopedia of Electrical and Electronics Engineering*, Wiley, Sept. 2015.

[4] B.-N. Vo, B.-T. Vo, N.-T. Pham and D. Suter, "Joint Detection and Estimation of Multiple Objects from Image Observations," *IEEE Trans. Signal Process.*, vol. 58, no.10, pp. 5129-5241, 2010.

[5] R. Mahler, *Statistical Multisource-Multitarget Information Fusion*, Artech House, 2007.

[6] R. Mahler, *Advances in Statistical Multisource-Multitarget Information Fusion*, Artech House, 2014.

[7] C. -Y. Chong, S. Mori, D. Reid, "Forty years of Multiple Hypothesis Tracking - A review of key developments,"*Proc. 21th Int. Conf. Inform. Fusion*, July, 2018.

[8] R. Mahler, "Multitarget Bayes filtering via first-order multitarget moments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 4, pp.1152–1178, 2003.

[9] B.-N. Vo, S. Singh, and A. Doucet, "Sequential Monte Carlo methods for multitarget filtering with random finite sets," *IEEE Trans. Aerosp. Electron. Syst.*, vol.41, no.4, pp. 224–1245, 2005.

[10] R. Mahler, "PHD Filters of Higher Order in Target Number," *IEEE Trans. Aerosp. Electron. Syst*, vol. 43, no. 3, pp. 1523-1543, 2007.

[11] B.-T. Vo, B.-N. Vo, and A. Cantoni, "Analytic implementations of the cardinalized probability hypothesis density filter," *IEEE Trans. Signal Process.*, vol. 55 no. 7, pp. 3553–3567, 2007.

[12] B.-T. Vo, B.-N. Vo, and A. Cantoni, "The Cardinality Balanced Multitarget Multi-Bernoulli filter and its implementations," *IEEE Trans. Signal Process.*, vol. 57, no. 2, pp. 409–423, 2009.

[13] B.-T. Vo and B.-N. Vo, "Labeled random finite sets and multi-object conjugate priors," *IEEE Trans. Signal Process.*, vol. 61, no. 13, pp. 3460–3475, 2013.

[14] B.-N. Vo, B.-T. Vo, and D. Phung, "Labeled random finite sets and the Bayes multitarget tracking filter," *IEEE Trans. Signal Process.*, vol. 62, no. 24, pp. 6554–6567, 2014.

[15] S. Reuter, B.-T. Vo, B.-N. Vo, and K. Dietmayer, "The labelled multi-Bernoulli filter," *IEEE Trans. Signal Process.*, vol. 62, no. 12, pp. 3246–3260, 2014.

[16] B.-T. Vo and B.-N. Vo, "Multi-Scan Generalized Labeled Multi-Bernoulli Filter," *Proc. 21st Int. Conf. Inform. Fusion*, July, 2018.

[17] B.-N. Vo, N. Dam, D. Phung, Q.N. Tran, and B.-T. Vo, "Model-Based Learning for Point Pattern Data," *Pattern Recognition*, https://doi.org/10.1016/j.patcog.2018.07.008, 2018.

[18] E. Maggio, M. Taj, and A. Cavallaro, "Efficient multitarget visual tracking using random finite sets," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 18, no. 8, pp. 1016–1027, 2008.

[19] R. Hoseinnezhad, B.-N. Vo, B. T. Vo, and D. Suter, "Visual tracking of numerous targets via multi-bernoulli filtering of image data," *Pattern Recognition*, vol. 45, no. 10, pp. 3625–3635, 2012.

[20] R. Hoseinnezhad, B.-N. Vo, and B.-T. Vo, "Visual tracking in background subtracted image sequences via multi-bernoulli filtering," *IEEE Trans. Signal Process.*, vol. 61, no. 2, pp. 392–397, 2013.

[21] J. Mullane, B.-N. Vo, M. Adams, and B.-T. Vo, "A random-finite-set approach to Bayesian SLAM," *IEEE Trans. Robot.*, vol. 27, no.2, pp. 268–282, 2011.

[22] D. Meissner, S. Reuter, and K. Dietmayer, "Road user tracking at intersections using a multiple-model PHD filter," *Proc. 2013 IEEE Intelligent Vehicles Symposium*, pp. 377–382, June 2013.

[23] B. Ristic, B.-N. Vo, and D. Clark, "A note on the reward function for PHD filters with sensor control," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 47, no. 2, pp. 1521–1529, 2011.

[24] H. G. Hoang and B. T. Vo, "Sensor management for multi-target tracking via multi-Bernoulli filtering," *Automatica*, vol. 50, no. 4, pp. 1135–1142, 2014.

[25] A. Gostar, R. Hoseinnezhad, and A. Bab-Hadiashar, "Robust multi-bernoulli sensor selection for multitarget tracking in sensor networks," *IEEE Signal Process. Lett.*, vol. 20, no. 12, pp. 1167–1170, 2013.

[26] H. Hoang, B.-N. Vo, B.-T., Vo, and R. Mahler, "The Cauchy-Schwarz divergence for Poisson point processes," *IEEE Trans. Inf. Theory*, vol. 61, no. 8, pp. 4475- 4485, 2015.

[27] A.K. Gostar, R. Hoseinnezhad, and A. Bab-Hadiashar, "Multi-Bernoulli sensor-selection for multitarget tracking with unknown clutter and detection profiles," *Signal Processing*, vol. 119, pp. 28-42, 2016.

[28] A.K. Gostar, R. Hoseinnezhad, Weifeng Liu, and A. Bab-Hadiashar, "Sensor-management for multitarget filters via minimization of posterior dispersion," *IEEE Trans. Aerosp. Electron. Syst.*, (in press) 2017.

[29] G. Battistelli, L. Chisci, C. Fantacci, A. Farina, and A. Graziano, "Consensus CPHD filter for distributed multitarget tracking," *IEEE J. Sel. Topics Signal Process.*, vol. 7, no. 3, pp. 508–520, 2013.

[30] B.-N. Vo, B.-T. Vo, and H. Hoang, "An Efficient Implementation of the Generalized Labeled Multi-Bernoulli Filter," *IEEE Trans. Signal Process.*, vol. 65, no. 8, pp. 1975–1987, 2017.

[31] M. Beard, B.-T. Vo, and B.-N. Vo, "Bayesian multitarget tracking with merged measurements using labelled random finite sets," *IEEE Trans. Signal Process.*, vol. 63, no. 6, pp. 1433-1447, 2015.

[32] M. Beard, S. Reuter, K. Granström, B.-T. Vo, B.-N. Vo, and A. Scheel, "Multiple extended target tracking with labeled random finite sets," *IEEE Trans. Signal Process.*, vol. 64, no. 7, pp. 1638- 1653, 2016.

[33] W. Yi, M. Jiang, and R. Hoseinnezhad, "The Multiple Model Vo-Vo Filter," *IEEE Trans. Aerosp. Electron. Syst.* Published online, DOI: 10.1109/TAES.2017.2667300, February 2017.

[34] F. Papi and D. Y. Kim, "A particle multitarget tracker for superpositional measurements using labeled random finite sets," *IEEE Trans. Signal Process.* vol. 63, no. 16, pp. 4348-4358, 2015.

[35] F. Papi, B.-N. Vo, B.-T. Vo, C. Fantacci, M. Beard, "Generalized Labeled Multi-Bernoulli Approximation of Multi-Object Densities," I*EEE Trans. Signal Process.*, vol. 63, no. 20, pp. 5487-5497, 2015.

[36] S. Li, W. Yi, R. Hoseinnezhad, B. Wang, L. Kong, "Multi-object Tracking for Generic Observation Model Using Labeled Random Finite Sets,"*IEEE Trans. Signal Process.*, vol. 66, no. 2, pp. 368–383, 2018.

[37] Y.G. Punchihewa, B.-T. Vo, B.-N. Vo, and D. Y. Kim, "Multiple Object Tracking in Unknown Backgrounds with Labeled Random Finite Sets," EEE Trans. Signal Processing, vol. 66, no. 11, pp. 3040-3055, 2018.

[38] D.Y. Kim, B.-N. Vo, and B.-T. Vo, "Online Visual Multi-Object Tracking via Labeled Random Finite Set Filtering," arXiv preprint arXiv:1611.06011, 2016.

[39] M. Beard, B.-T. Vo, B.-N. Vo, and S. Arulampalam, "Void probabilities and Cauchy-Schwarz divergence for Generalized Labeled Multi-Bernoulli Models," *IEEE Trans. Signal Process.*, vol. 65, no. 19, pp. 5047–5061, 2017.

[40] X. Wang, R. Hoseinnezhad, A. K. Gostar, T. Rathnayake, B. Xu, A. Bab-Hadiashar, "Multi-Sensor Control for Multi-Object Bayes Filters, "*Signal Processing*, vol.142, pp. 260–270, 2018.

[41] C. Fantacci, B.-N. Vo, B.-T. Vo, G. Battistelli, and L. Chisci, "Robust fusion for multisensor multiobject tracking" *IEEE Signal Process. Lett.*, vol. 25, no. 5, pp. 640–644, 2018.

[42] S. Li, W. Yi, R. Hoseinnezhad, G. Battistelli, B. Wang, L. Kong, "Robust Distributed Fusion with Labeled Random Finite Sets, " *IEEE Trans. Signal Process.*, vol. 66, no. 2, pp. 278– 293, 2018.

[43] H. Deusch, S. Reuter, and K. Dietmayer, "The labeled multi-Bernoulli SLAM filter," *IEEE Signal Process. Lett.*, vol. 22, no. 10, pp. 1561-1565, 2015.

[44] W.J. Hadden et. al. "Stem cell migration and mechanotransduction on linear stiffness gradient hydrogels," *Proc. National Academy of Sciences*, vol. 114, no. 22, pp. 5647-5652, 2017.

[45] M. Tobias, A. Lanterman, "A probability hypothesis density-based multitarget tracker using multiple bistatic range and velocity measurements," in Proc. *36th Southeastern Symp. System Theory*, 2004.

[46] L. Ma, D. Y. Kim, and X. Kai, "Multi-Bernoulli filter for target tracking with multistatic Doppler only measurement," *Signal Processing*, vol. 108, pp. 102-110, 2015.

[47] M. Beard, B. -T. Vo, B. -N. Vo, "A Solution for Large-scale Multi-object Tracking " *arXiv preprint arXiv* :1804.06622, 2018.

[48] F. Guo,Y. Fab, Y. Zhou, C. Xhou, Q. Li "Space electronic reconnaissance: localization theories and methods, "*John Wiley & Sons*, 2014.

[49] B.-N. Vo, and B.-T. Vo, "An implementation of the multi-sensor generalized labeled multi-Bernoulli filter via Gibbs sampling," *Proc. 20th Int. Conf. Inform. Fusion*, pp. 1-8, July, 2017.

[50] B.S. Wei, B. Nener, W.F. Liu; and M. Liang, "Centralized Multisensor Multitarget Tracking with Labeled Random Finite Sets," *Proc. Int. Conf. Control, Aut. & Inf. Sciences,* 2016.

[51] Schuhmacher, Dominic, B. -T. Vo, and B. -N. Vo "A consistent metric for performance evaluation of multi-object filters, "*IEEE Transactions on Signal Processing* vol. 56, no. 8, pp. 3447-3457, 2008.

[52] X. Wang, A. K. Gostar, T. Rathnayake, B. Xu, A. Bab-Hadiashar and R. Hoseinnezhad, "Centralized Multiple-View Sensor Fusion Using Labeled Multi-Bernoulli Filters, " *IEEE Trans. Signal Process.* vol.150, pp. 75 – 84, 2018.

[53] W. Yi, M. Jiang, R. Hoseinnezhad, B. Wang, "Distributed Multi-Sensor Fusion Using Generalized MultiBernoulli Densities, "*IET Radar, Sonar & Navigation*, vol. 11, no. 3, pp. 434– 443, March 2017.