Label Management for Multi-Object Estimates via Labelled Multi-Bernoulli Filters

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***Abstract*—This work addresses the challenging problem of online managing labels from a sequence of unlabelled multi-object estimates, which is a crucial task in multi-object tracking, particularly in noisy environments with unknown and time-varying numbers of mobile objects. The considered problem requires solving a multi-dimensional assignment problem, which is an NP-hard problem. In this paper, we propose reformulating the aforementioned label management problem within the recursive Bayesian framework and leveraging the effectiveness of the labelled multi-Bernoulli (LMB) filter to accurately and efficiently assign labels to unlabelled multi-object estimates in real-time. The proposed LMB labelling (LMBL) algorithm is *agnostic* to the filtering method and is capable of labelling any unlabelled multi-object estimates. Experimental results demonstrate significant labelling performance improvements of our proposed LMBL approach compared to other state-of-the-art methods. This work presents a robust and efficient solution for the critical problem of label management in multi-object tracking applications.**

***Index Terms*—Label Management, Re-ID, Point Correspondences, RFS, LMBL.**

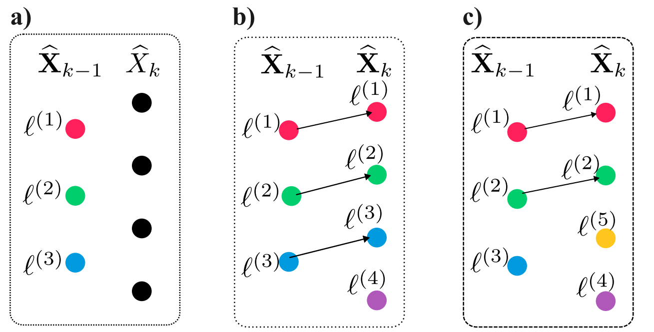

Fig. 1. a) **An example of a label management problem:** How to assign labels for a multi-object estimate $\widehat{X}_k$ given the labelled multi-object estimate $\widehat{\mathbf{X}}_{k-1}$. **b) One possible hypothesis:** all objects survive from time $k-1$ to time k, and one object is born at time k with new label $\ell^{(4)}$. **c) Another possible hypothesis:** two objects ($\ell^{(1)}$ and $\ell^{(2)}$) survive from time $k-1$ to k, while object $\ell^{(3)}$ dies, and two objects ($\ell^{(4)}$ and $\ell^{(5)}$) are born at time k. We propose using the LMB filter to solve this problem, which considers all possible hypotheses, including hypotheses (b) and (c).

I. Introduction

Multiple Object Tracking (MOT) addresses the complex challenge of simultaneously estimating the time-varying number of objects (due to random appearances/disappearances) and their corresponding trajectories from noisy measurements. This process is further complicated by false alarms, incorrect detections, and uncertain data associations (i.e., ambiguous measurement-to-object origins). MOT is a mature field that has evolved over six decades since the 1960s, finding applications across diverse domains such as surveillance [1], search and rescue operations [2], [3], autonomous driving [4], robotics [5], [6], remote sensing [7], star tracking [8], computer vision [9], [10], aerospace [11], and biomedical sensing [12], [13].

Although there are several approaches to solving MOT problems, most algorithms can be categorised into one of the following paradigms: i) Multiple Hypothesis Tracking (MHT) [11], ii) Joint Probabilistic Data Association (JPDA) [14], [15] and iii) Random Finite Set (RFS) [16], [17]. MHT and JPDA are traditional MOT approaches that first solve the data association and then employ a single-object filter (e.g., Kalman Filter). In contrast, the RFS framework, first introduced in 1997 in [18] and explained in detail in [16], [17], considers the multi-object state as a finite set and aims to find the optimal/sub-optimal estimation of a multi-object state instead of concentrating on solving the data-association problem. Some RFS-based algorithms, such as Probability Hypothesis Density (PHD) [19] and Cardinalised Probability Hypothesis Density (CPHD) [20] filters, do not involve solving data association at all, which significantly improves the processing time.

Since its introduction in 1997 [18], the RFS-based framework has also considered including the object's identity/label in the state, which is crucial for acquiring interactions amongst multiple objects and obtaining individualised object-related information (e.g., trajectories). However, most initial RFS-based filters (e.g., PHD, CPHD, MB, PMBM) [20]–[23] omitted the object labels. The labelled RFS theory, proposed in 2011 [24], yielded a mathematically rigorous formulation to realise the first Bayes-optimal MOT algorithm, namely the Generalised Labelled Multi-Bernoulli (GLMB) filter in 2013 [25], [26]. The GLMB filter has been effectively implemented using Gibbs sampling via a joint prediction-update procedure [27], and further simplified with a linear complexity using tempered Gibbs sampling technique [28]. An alternative computationally efficient filter based on labelled Random Finite Sets (RFS) is the Labelled Multi-Bernoulli (LMB) filter [29]. This approach systematically approximates the Generalised Labelled Multi-

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Bernoulli (GLMB) filter by aligning its first-order statistical moment.

One of the key challenges in MOT is the unknown data association problem, where the number of association hypotheses increases exponentially over time. Finding the optimal set of association hypotheses is a computationally expensive task, often requiring efficient algorithms such as Murty's algorithm [30], [31] or Gibbs sampling techniques [27].

Alternatively, efficient MOT algorithms can be realised by solving the data association problems directly on multi-object estimates, which is equivalent to the re-identification (re-ID) or Label Management problem (as illustrated in Fig. 1). A significant number of recent re-ID methods are based on bipartition matching between two scans or frames (e.g., using the Hungarian algorithm) leveraging spatial information [32], [33] and/or deep learning features [34], [35]. Multi-frame matching via a graph was also proposed in [36], and later applied for unlabelled RFS filters in [37].

In this work, we realise a new and effective algorithm to solve MOT by performing label assignments between two scans using the LMB filter on multi-object estimates. The proposed method is *agnostic* to filter types and is capable of labelling the non-labelled multi-object estimates from any non-labelled MOT filters (e.g., PHD, CPHD filters). The developed algorithm utilises the robustness of LMB filters to assign correct labels to multi-object estimates via a soft-decision process in LMB filters, thereby reducing label switching errors due to misdetections compared to traditional methods [36].

The remainder of this work is organised as follows. Section II provides important information on labelled RFSs and their corresponding filters. The realised label management algorithm, utilising the LMB filter for multi-object estimates, is shown in Section III. Section IV presents experiments and comparisons with baseline methods. Section V concludes our observations.

II. Background

A. Notations

In this work, we adopt the conventional notations used in [26]. Lowercase letters (e.g., $x, \mathbf{x}$) are used to represent individual object states, whereas uppercase letters (e.g., $X, \mathbf{X}$) denote multi-object states. Boldfaced letters (e.g., $\mathbf{x}, \mathbf{X}, \boldsymbol{\pi}$) are used to denote labelled object states and labelled multi-object densities. Blackboard letters such as $\mathbb{L}, \mathbb{X}, \mathbb{Z}$ denote spaces. The class of all finite subsets of X is denoted by $\mathcal{F}(X)$. The inner product of two functions is represented as $\langle h, g\rangle = \int h(x)g(x)dx$, and the multi-object exponential is given by $f^X = \prod_{x\in X} f(x)$, with $f^{\emptyset} = 1$. The number of members of a set X is represented by $|X|$. To accommodate different types of arguments (e.g., sets, vectors), we define the Kronecker delta function $\delta_Y(Z) = 1$ if $Z = Y$ else $\delta_Y(Z) = 0$, and indicator function $1_Y(Z) = 1$ if $Z \subseteq Y$ else $1_Y(Z) = 0$.

B. Random Finite Sets

Random finite sets (RFSs) are set-valued random variables [16]. The members of an RFS are characterised by their stochastic nature and lack of inherent order, making the RFS an ideal framework for naturally representing multi-object states. A labelled RFS can be considered as an extension of the standard RFS, where each element is assigned a unique identifier or label [25]. Employing Mahler's Finite Set Statistics (FISST) methodology, an RFS can be comprehensively characterised by its corresponding FISST density function. In this paper, we present and utilise three different types of Random Finite Sets: Bernoulli RFS, Labelled Multi-Bernoulli (LMB) RFS and Generalised Labelled Multi-Bernoulli (GLMB) RFS.

1) Bernoulli RFS: The Bernoulli RFS is one of the simplest types of RFS and serves as a fundamental building block for more complex RFS models. It is beneficial for representing the state of a single object that may or may not exist. Two parameters characterise a Bernoulli RFS X are: i) r is the probability of existence ($0 < r < 1$); and ii) $p(\cdot)$ is the probability density of the single object's state, given that it exists. The density $\pi(\cdot)$ of a Bernoulli RFS is given by

$$\pi(X) = \begin{cases} rp(x), & X = \{x\} \\ 1 - r, & X = \emptyset \end{cases}. \tag{1}$$

Multi-Bernoulli RFS is a union of multiple Bernoulli RFSs.

2) LMB RFS: The LMB RFS extends the Multi-Bernoulli RFS by incorporating unique labels for each object. It is defined by a set of parameters $\{r^{(\ell)}, p^{(\ell)}\}_{\ell\in\mathbb{L}}$, where $r^{(\ell)}$ denotes the existence probability of an object with label ℓ, and $p^{(\ell)}$ represents its corresponding spatial density if the object ℓ exists. The density $\boldsymbol{\pi}$ of an LMB RFS is expressed as [29]

$$\boldsymbol{\pi}(\mathbf{X}) = \triangle(\mathbf{X})w(\mathcal{L}(\mathbf{X}))p^{\mathbf{X}}, \tag{2}$$

where $\triangle(\mathbf{X}) = \delta_{|\mathbf{X}|}(\mathcal{L}(|\mathbf{X}|))$ ensures unique labels, with $\mathcal{L}(\mathbf{X})$ representing labels extracted from $\mathbf{X}$; $w(L) = (1-r)^{\mathbb{L}\setminus L}r^L$; and $p(\mathbf{x}) = p(x, \ell) = p^{(\ell)}(x)$.

3) GLMB RFS: The GLMB RFS represents a more general form of the LMB RFS. It is described as a combination of multi-object exponentials, each corresponding to a distinct combination of object labels, or a hypothesis. The GLMB is completely characterised by its density function, which is expressed as [25], [26]:

$$\boldsymbol{\pi}(\mathbf{X}) = \triangle(\mathbf{X})\sum_{c\in\mathbb{C}} w^{(c)}(\mathcal{L}(\mathbf{X}))[p^{(c)}]^{\mathbf{X}}, \tag{3}$$

where $\mathbb{C}$ is a discrete set. In practice, the GLMB density is often represented under the δ-GLMB form, where $\mathbb{C} = \Xi \times \mathcal{F}(\mathbb{L})$ with Ξ is a discrete space containing all association histories, $w^{(c)}(L) = w^{(\xi,J)}\delta_J(L)$, and $p^{(c)} = p^{(\xi,J)} = p^{(\xi)}$.

C. Multi-Object Tracking via RFS Framework

In a multi-object system, akin to a traditional dynamical system where the system state is represented by a time-evolving vector, the system state is instead a finite set. To model the uncertainty in this multi-object state, an RFS is utilised, much like a random vector models uncertainty in a state vector. Given the measurement history $Z_{1:k} = (Z_1, \dots, Z_k)$ from the initial time to time k, the density of multi-object states $\mathbf{X}$ can be recursively propagated from an initial prior $\boldsymbol{\pi}_0(\mathbf{X})$ using Mahler's set integral (FISST) through Bayesian recursion [16]:

$$\boldsymbol{\pi}_{k|k-1}(\mathbf{X}_k) = \int \boldsymbol{\pi}_{k-1}(\mathbf{X})\mathbf{f}_{k|k-1}(\mathbf{X}_k|\mathbf{X})\delta\mathbf{X}, \tag{4}$$

$$\boldsymbol{\pi}_k(\mathbf{X}_k|Z_k) = \frac{\boldsymbol{\pi}_{k|k-1}(\mathbf{X}_k)g(Z_k|\mathbf{X}_k)}{\int \boldsymbol{\pi}_{k|k-1}(\mathbf{X})g(Z_k|\mathbf{X})\delta\mathbf{X}}, \tag{5}$$

where $\mathbf{f}_{k|k-1}(\cdot|\cdot)$ represents the multi-object transition density from time $k-1$ to time k, incorporating the dynamics of object survival, birth, and death; The function $g(Z_k|\cdot)$ represents the multi-object observation likelihood at time k, incorporating false alarms, misdetections, and uncertainties in data association.

1) GLMB Filter: In a typical multi-object state space system, the GLMB RFS maintains its conjugacy property and is preserved via Bayesian recursion [25], as described by equations (4) and (5). Consequently, the Bayes recursion transforms a GLMB multi-object density at time $k-1$ into a GLMB density at time k, given by:

$$\boldsymbol{\pi}_k = \varOmega(\boldsymbol{\pi}_{k-1}, Z_k), \tag{6}$$

where $\varOmega$ represents the GLMB Bayes recursive operator (see [27] for detailed expressions of $\varOmega$). Thus, beginning with an inaugural GLMB RFS, all successive multi-object densities are GLMB RFSs. However, the uncertainties of data association cause the number of hypotheses within the GLMB density to grow super-exponentially over time, requiring truncation to control this growth.

2) LMB Filter: The LMB RFS does not remain closed under the Bayes update in (5). This is because an initial LMB density transforms into a GLMB density after the update step, rather than remaining an LMB [29]. Consequently, to implement the LMB filter, the updated GLMB RFS must be approximated as an LMB RFS by preserving its first moment, denoted by the transformation T. Given the updated GLMB $\boldsymbol{\pi} = \{(w^{(\xi)}(J), p^{(\xi)}) : (\xi, J) \in \varXi \times \mathcal{F}(\mathbb{L})\}$, its LMB first moment approximation is $T(\boldsymbol{\pi}) = \{r^{(\ell)}, p^{(\ell)}\}_{\ell\in\mathbb{L}}$, where:

$$r^{(\ell)} = \sum_{\xi,J} 1_J(\ell) w^{(\xi)}(J), \tag{7}$$

$$p^{(\ell)}(x) = \frac{1}{r^{(\ell)}} \sum_{\xi,J} 1_J(\ell) w^{(\xi)}(J) p^{(\xi)}(x, \ell). \tag{8}$$

In essence, the LMB filter involves two steps: i) propagate the LMB density from the current time to the next time step via GLMB recursion as per (6), and ii) approximate the resulting GLMB density as an LMB density, such that:

$$\boldsymbol{\pi}_k = T(\varOmega(\boldsymbol{\pi}_{k-1}, Z_k)). \tag{9}$$

III. Label Management Method

A. Problem Statement

The core challenge addressed in this paper is the labelling of multi-object state estimates in a dynamic environment. Given a sequence of unlabeled multi-object state estimates $\widehat{X}_{1:k} = (\widehat{X}_1, \ldots, \widehat{X}_k)$ from the initial time to the current time k, our objective is to assign appropriate labels $L_{1:k} = (L_1, \ldots, L_k)$ to $\widehat{X}_{1:k}$, thereby realising labelled multi-object estimates $\widehat{\mathbf{X}}_{1:k}$, where $L_k = \mathcal{L}(\widehat{\mathbf{X}}_k)$.

Solving this problem as a multi-dimensional assignment problem (with dimension k increasing over time) is computationally intractable. Therefore, we propose a tractable recursive approach. The problem can be reformulated as follows:

Given:

- A set of labelled multi-object state estimates $\widehat{\mathbf{X}}_{k-1} = \{(x_{k-1}^{(1)}, \ell_{k-1}^{(1)}), \ldots, (x_{k-1}^{(N)}, \ell_{k-1}^{(N)})\}$ at the previous time step $k-1$.
- An unlabeled multi-object state estimate $\widehat{X}_k = \{x_k^{(1)}, \ldots, x_k^{(M)}\}$ at the current time step k.

The task is to assign appropriate labels $L_k = \{\ell_k^{(1)}, \ldots, \ell_k^{(M)}\}$ to $\widehat{X}_k$ (illustrated in Fig. 1).

It is crucial to recognise that the number of objects may fluctuate between time steps due to birth and death processes as well as false alarms and misdetections. When assigning labels, we must consider two scenarios:

- Objects at time k are survivors from time $k-1$: These objects should retain their previous labels,
- Objects at time k are newly born: These objects should be assigned new, unique labels.

Traditional approaches, such as bi-partition matching [32], attempt to solve this problem through hard decision processes. However, these methods often fall short in accounting for complexities like misdetections, false alarms, and birth/death processes.

In this work, we propose a novel approach using a Labelled Multi-Bernoulli (LMB) filter. This method incorporates labels directly into the state estimates, providing a more robust and flexible solution to the multi-object labelling problem. Our approach can handle the uncertainties and dynamics inherent in MOT scenarios, offering a significant improvement over existing techniques.

B. Label Management using LMB Filters

Proposed Approach: In this work, we propose considering the multi-object estimate[1] $\widehat{X}_k$ as a measurement Z_k at time k with a single-object measurement likelihood $g(z|x) = \mathcal{N}(z; x, R)$ where $\mathcal{N}(\cdot; \mu, \varSigma)$ represents a Gaussian density with μ and $\varSigma$ are its mean and covariance, respectively. In this work, we assume the motion model $f(\cdot|\cdot)$ of objects is known and is equivalent to the original state space model used to obtain the non-labelled estimates. Additionally, we propose using the measurement-driven adaptive birth procedure [29] to initialise the LMB birth distribution. They are detailed as follows.

Prior: At initial time ($k = 0$), the initial LMB density $\boldsymbol{\pi}_0 = \emptyset$. Using the recursive LMB filter in (9) for time 0 to $k-1$ with $Z_i \equiv \widehat{X}_i$ for $i = 1, \ldots, k-1$, which yields $\boldsymbol{\pi}_{k-1}$.

[1] The estimate $\widehat{X}_k$ is a realisation of multi-object state of X_k.

Prediction: At current time k, the predicted LMB density comprises the surviving density $\boldsymbol{\pi}_{S,k}$ of labelled tracks from time $k-1$, as well as the adaptive birth density $\boldsymbol{\pi}_{B,k}$.

$$\boldsymbol{\pi}_{k|k-1} = \boldsymbol{\pi}_{S,k} \cup \boldsymbol{\pi}_{B,k}. \quad (10)$$

Here, using the measurement-driven birth procedure and to prevent data incest, the birth density $\boldsymbol{\pi}_{B,k}$ at time k depends on the set of measurements Z_{k-1} at the previous time $k-1$, given by:

$$\boldsymbol{\pi}_{B,k} = \left\{ r_{B,k}^{(\ell)}(z), p_{B,k}^{(\ell)}(x|z) \right\}_{\ell \in \mathbb{B}_k}, \quad (11)$$

where $\mathbb{B}_k = \{(k,1), \ldots, (k, |Z_{k-1}|)\}$ and

$$r_{B,k}^{(\ell)}(z) = \min\left(r_{B,\max}, \frac{\lambda_{B,k}(1 - r_U(z))}{\sum_{\zeta \in Z_{k-1}} (1 - r_U(\zeta))} \right). \quad (12)$$

In this context, $r_U(z)$ represents the probability of associating the measurement z with one of the objects, as detailed in Equation (73) of [29]. The term $\lambda_{B,k}$ denotes the expected number of new births. For this study, we set $\lambda_{B,k} = \max(0, M - N)$, implying that if the number of estimates at time k is less than or equal to N, no new objects (i.e., labels) are introduced.

Update: Given the predicted LMB density $\boldsymbol{\pi}_{k|k-1}$ and the measurement $Z_k \equiv \widehat{X}_k$, the posterior GLMB density is computed via (5):

$$\boldsymbol{\pi}_k^{(\text{glmb})}(\mathbf{X}_k) \propto g(Z_k|\mathbf{X}_k)\boldsymbol{\pi}_{k|k-1}(\mathbf{X}_k). \quad (13)$$

Here, the multi-object observation likelihood function is given by

$$g(Z|\mathbf{X}) \propto \sum_{\theta \in \Theta(\mathcal{L}(\mathbf{X}))} \left[\Psi_Z^{(\theta \circ \mathcal{L}(\cdot))}(\cdot) \right]^{\mathbf{X}}, \quad (14)$$

where $\Theta(J)$ is the set of association maps of J, and

$$\Psi_Z^{(i)}(\mathbf{x}) = \begin{cases} P_D(\mathbf{x}) \dfrac{g(z^{(i)}|x)}{\kappa(z^{(i)})}, & i \in 1, \ldots, |Z| \\ 1 - P_D(\mathbf{x}), & i = 0 \end{cases}. \quad (15)$$

Here, $\kappa(\cdot)$ denotes the clutter density which follows a Poisson distribution with a clutter rate of $\langle 1, \kappa \rangle$. The posterior GLMB density is approximated as an LMB density via the operator T defined in (7) and (8): $\boldsymbol{\pi}_k = T(\boldsymbol{\pi}_k^{(\text{glmb})})$.

Modified LMB Estimator: Given the posterior LMB density $\boldsymbol{\pi}_k = \{r_k^{(\ell)}, p_k^{(\ell)}\}_{\ell \in \mathbb{L}_k}$, one needs to use an estimator to compute the labelled multi-object estimate $\widehat{\mathbf{X}}_k$ from $\boldsymbol{\pi}_k$. In this work, we have an advantage compared to the standard LMB filter that the number $|\widehat{\mathbf{X}}_k|$ of multi-object estimates is known and equal to the cardinality of $|Z_k|$ since we consider $Z_k \equiv \widehat{X}_k$. Additionally, since $Z_k \equiv \widehat{X}_k$, we propose constructing an additional $\boldsymbol{\pi}_{A,k}$ LMB density from Z_k (equivalent the birth density $\boldsymbol{\pi}_{B,k+1}$ at the next time step), defined as:

$$\boldsymbol{\pi}_{A,k} = \left\{ r_A^{(\ell)}(z), p_A^{(\ell)}(x|z) \right\}_{\ell \in \mathbb{B}_{k+1}}, \quad (16)$$

where $\mathbb{B}_{k+1} = \{(k+1,1), \ldots, (k+1, |Z_k|)\}$ and

$$r_A^{(\ell)}(z) = \min\left(r_{B,\max}, \frac{\lambda_{B,k+1}(1 - r_U(z))}{\sum_{\zeta \in Z_k} (1 - r_U(\zeta))} \right), \quad (17)$$

and $\lambda_{B,k+1} = \min(0, |Z_k| - |Z_{k-1}|)$.

Let $\tilde{\boldsymbol{\pi}}_k$ be the modified LMB density, given by

$$\tilde{\boldsymbol{\pi}}_k = \boldsymbol{\pi}_k \cup \boldsymbol{\pi}_{A,k} = \left\{ \tilde{r}_k^{(\ell)}, \tilde{p}_k^{(\ell)} \right\}_{\ell \in \mathbb{L}_k \cup \mathbb{B}_{k+1}}, \quad (18)$$

and $\tilde{L}_k$ be the list of $|Z_k|$ labels with the highest existence probability in $\tilde{\boldsymbol{\pi}}_k$. The labelled multi-object estimate $\widehat{\mathbf{X}}_k$ can be computed from $\tilde{\boldsymbol{\pi}}_k$ using the maximum a posterior (MAP) method, given by:

$$\widehat{\mathbf{X}}_k = \left\{ (x, \ell) : x = \int y \tilde{p}_k^{(\ell)}(y) dy, \ell \in \tilde{L}_k \right\}. \quad (19)$$

IV. Experiments

We demonstrate the performance of our proposed label management algorithm (**LMBL**) via simulated experiments in MATLAB. We consider two different baseline algorithms: i) the bi-partition matching labelling algorithm [32] (denoted as **BML**) and ii) the multi-frame graph labelling algorithm [36] (denoted as **MGL**). For **MGL**, the number of frames used for label matching is 5. Experiments were conducted on a workstation with two EPYC 7702 Processors @ 2.0 GHz and 1024 GB of memory. All results were computed as the average over 100 Monte Carlo (MC) runs. We utilise labelling computational time, the optimal sub-pattern assignment (OSPA) metric [38], and the OSPA-on-OSPA (OSPA$^{(2)}$) metric [39] to evaluate the labelling performance of these algorithms. A smaller OSPA$^{(2)}$ value indicates superior tracking performance regarding localisation accuracy and minimising track switching errors.

We conduct a 2D tracking experiment featuring a fluctuating number of objects due to the object's random appearances/disappearances with a total number of objects of 22. The entire simulation lasts for 100 s, with measurements taken at intervals of $\Delta = 1$ s. Fig. 2 presents the experimental setup, showcasing the ground truth trajectories of the 22 mobile objects.

Object Motion Model: The objects are described by a constant velocity model within a two-dimensional environment. Each individual state, $x = [p_x, \dot{p}_x, p_y, \dot{p}_y]^\top$, represents its kinematic state, with $\top$ indicating the transpose. The dynamic model is given by $f_{k|k-1}(x_k|x_{k-1}) = \mathcal{N}(x_k; Fx_{k-1}, Q)$. The state transition matrix F and the process noise covariance Q are defined as:

$$F = \begin{bmatrix} 1 & \Delta \\ 0 & \Delta \end{bmatrix} \otimes I_2; \quad Q = \sigma_v^2 \begin{bmatrix} \Delta^3/3 & \Delta^2/2 \\ \Delta^2/2 & \Delta \end{bmatrix} \otimes I_2$$

where $\otimes$ represents Kronecker product, I_2 is the identity matrix, and $\sigma_v = 5$ m/s^2 is the process noise standard deviation. The survival probability P_S of each object is 0.99.

Unlabelled Filter: To validate the effectiveness of our proposed label management algorithm, we employ the *CPHD filter* [20] to generate the unlabelled multi-object estimate

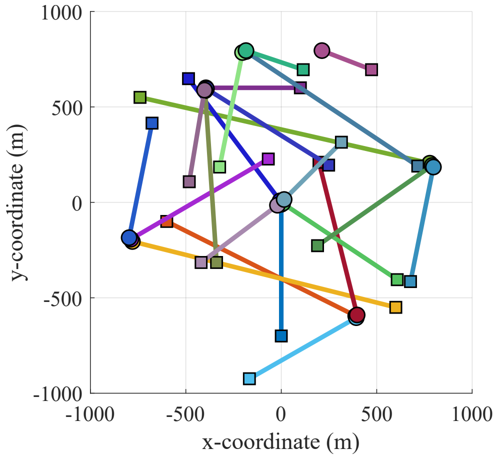

Fig. 2. **Ground truth trajectories** of 22 mobile objects in a 2D environment. We use ◯/□ to represent each object's starting/stopping locations. Different colours denote different object labels.

TABLE I
AVERAGED TRACKING PERFORMANCE OVER 100 MC RUNS

Methods	**OSPA [m]**	**OSPA$^{(2)}$ [m]**	**Label Time [ms]**
BML	18.13	45.03	**4.83**
MGL	18.13	42.69	155.78
LMBL (Ours)	**10.37**	**25.34**	30.44

$\widehat{X}_k$. For measurements of the CPHD filter, we utilise a 2D position sensor with a detection probability of $P_D = 0.95$. Each detected object x provides a noisy measurement $z = [z_x, z_y]^\top$. The likelihood of the measurement is defined as $g^{(\text{cphd})}(z|x) = \mathcal{N}(x; Gx, \sigma_r^2 I_2)$, where $G = [I_2, 0_2]$ and 0_2 is a zero matrix, with measurement noise $\sigma_r = 10$ m. Each measurement interval also includes clutters (false alarms) in the measurement set, with a clutter rate of 10, i.e., on average, there are 10 clutters per scan.

Results: Fig. 3 shows the **labelled** multi-object state estimates for a particular run using a) **BML**, b) **MGL**, and c) **LMBL**. Table I presents a detailed comparative analysis, averaged over 100 MC trials, between our proposed label management approach (**LMBL**) and the baseline methods.

The experimental outcomes validate that our labelling algorithm significantly outperforms **BML** and **MGL** in terms of OSPA and OSPA$^{(2)}$, while only slightly slower than the **BML** method, which is expected since BML is the simplest method and does not consider the joint probabilistic association when assigning new labels. As a result, **BML** yields the worst performance with a substantial amount of track switching (see Fig. 3a). **MGL** performs slightly better than **BML** (see Fig. 3b) since it uses multi-frame information to resolve labels; however, **MGL** requires significant labelling time for solving the multi-dimensional assignment problem across multiple frames. Additionally, **MGL** is susceptible to misdetections resulting in several label-switching errors. In contrast, the proposed **LMBL** method yields the best labelling result with minimal label switching errors as shown in Fig. 3c.

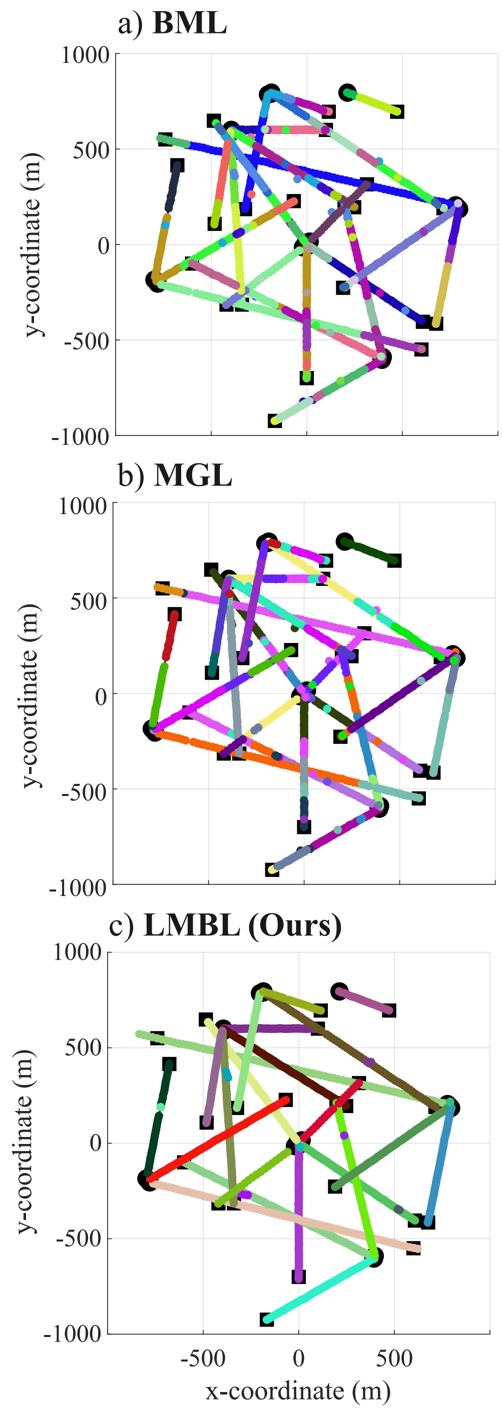

Fig. 3. The **labelled** estimated trajectories from **unlabelled** estimates $\widehat{X}_{1:100}$ for a particular run using a) Bi-partition Matching Labelling (**BML**), b) Multi-frame Graph Labelling (**MGL**), and c) Our proposed LMB Labelling (**LMBL**). Each colour denotes each estimated object label.

V. CONCLUSION

We have developed an online, efficient and robust label management algorithm for unlabelled multi-object estimates. Our solution employs a novel approach where multi-object estimates are treated as a measurement set, and a modified LMB filter is used to assign labels to these *unlabelled* estimates.

The proposed method is *agnostic* to filter types, enabling it to label unlabelled multi-object estimates from any non-labelled multi-object tracking (MOT) filters. Experimental results highlight the robustness of our **LMBL** method, demonstrating its effectiveness in labelling a sequence of *unlabelled* multi-object estimates generated from a CPHD filter. The improved accuracy of **LMBL** is attributed to the soft-decision process in LMB filters, which is more effective than the hard-decision labelling process employed in the baseline methods. We envisage that the proposed method can be extended with the smooth estimator [40], or multi-sensor and/or multi-scan approaches [41]–[43].

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